Formelsammlung für Finanzmathematik 2
Manfred Jäger-Ambrozewicz, www.mathfred.de

EPFMM

Anschaffungskosten EPFMM

$$V_0^{\mathbf{h}} = h_0 + S_0^1 h_1 + ...S_0^N h_N \quad (1)$$
$$= (1,\ \mathbf{S}_0^T)\,\mathbf{h} \quad (2)$$
$$= \mathbf{h}^T \begin{pmatrix} 1 \\ S_0^1 \\ \vdots \\ S_0^N \end{pmatrix} \quad (3)$$
$$= \mathbf{h}^T \begin{pmatrix} 1 \\ \mathbf{S}_0 \end{pmatrix} = \langle \mathbf{h}, \begin{pmatrix} 1 \\ \mathbf{S}_0 \end{pmatrix} \rangle \quad (4)$$
$$= \langle \mathbf{h}, \bar{\mathbf{S}}_0 \rangle, \quad (5)$$

Auszahlung EPFMM

$$\mathbf{V}_1^{\mathbf{h}} = \mathbf{A}\mathbf{h}. \quad (6)$$

Auszahlungsmatrix

$$\mathbf{A} = \begin{pmatrix} 1+r & S_1^1(\omega_1) & ... & S_1^N(\omega_1) \\ 1+r & S_1^1(\omega_2) & ... & S_1^N(\omega_2) \\ \vdots & \vdots & \vdots & \vdots \\ 1+r & S_1^1(\omega_K) & ... & S_1^N(\omega_K) \end{pmatrix}.$$

Arbitrage

$$V_0^{\mathbf{h}} = 0, 0 \neq \mathbf{V}_1^{\mathbf{h}} \geq \mathbf{0} \quad (7)$$

bzw.

$$V_0^{\mathbf{h}} = 0, \mathbf{V}_1^{\mathbf{h}} \geq \mathbf{0} \text{ und} \quad (8)$$
$$V_1^{\mathbf{h}}(\omega) > 0 \text{ für mindestens ein } \omega \in \Omega. \quad (9)$$

Risikoneutralwahrscheinlichkeit

$$\mathbb{Q}(\omega) > 0 \quad (10)$$
$$S_0^i = \mathbb{E}^{\mathbb{Q}}\left(\frac{S_1^i}{1+r}\right) \quad (11)$$
$$= \sum_{k=1}^K q_k \frac{S_1^i(\omega_k)}{1+r} = \sum_{k=1}^K q_k \cdot S_1^{i*}(\omega_k) \quad (12)$$
$$\mathbf{S}_0 = (\mathbf{S}_1^*)^T \mathbf{q} \quad (13)$$
$$q_i = q(\omega_i) = \mathbb{Q}(\omega_i) \quad (14)$$

$$\mathbf{S}_1^* = \begin{pmatrix} \frac{S_1^1(\omega_1)}{1+r} & ... & \frac{S_1^{N-1}(\omega_1)}{1+r} & \frac{S_1^N(\omega_1)}{1+r} \\ \frac{S_1^1(\omega_2)}{1+r} & ... & \frac{S_1^{N-1}(\omega_2)}{1+r} & \frac{S_1^N(\omega_2)}{1+r} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{S_1^1(\omega_K)}{1+r} & ... & \frac{S_1^{N-1}(\omega_K)}{1+r} & \frac{S_1^N(\omega_K)}{1+r} \end{pmatrix} \quad (15)$$

Risikoneutralwahrscheinlichkeit

$$\mathbf{S}_0 = (\mathbf{S}_1^*)^T \mathbf{q}, \sum_{i=1}^K q_i = 1, \mathbf{q} > 0. \quad (16)$$
$$\bar{\mathbf{S}}_0 = (\mathbf{A}^*)^T \mathbf{q}, \mathbf{q} > 0 \quad (17)$$

$$\mathbf{A}^* = \begin{pmatrix} 1 & \frac{S_1^1(\omega_1)}{1+r} & ... & \frac{S_1^{N-1}(\omega_1)}{1+r} & \frac{S_1^N(\omega_1)}{1+r} \\ 1 & \frac{S_1^1(\omega_2)}{1+r} & ... & \frac{S_1^{N-1}(\omega_2)}{1+r} & \frac{S_1^N(\omega_2)}{1+r} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \frac{S_1^1(\omega_K)}{1+r} & ... & \frac{S_1^{N-1}(\omega_K)}{1+r} & \frac{S_1^N(\omega_K)}{1+r} \end{pmatrix} \quad (18)$$

Risikoneutralwahrscheinlichkeit

$$V_0^{\mathbf{h}} = \sum_{k=1}^K q(\omega_k) \frac{V_1^{\mathbf{h}}(\omega_k)}{1+r} \quad (19)$$
$$= \sum_{k=1}^K q(\omega_k)(V_1^{\mathbf{h}})^*(\omega_k) \quad (20)$$
$$= ((\mathbf{V}_1^{\mathbf{h}})^*)^T \mathbf{q} = \mathbf{q}^T (\mathbf{V}_1^{\mathbf{h}})^* \quad (21)$$
$$= \langle \mathbf{q}, (\mathbf{V}_1^{\mathbf{h}})^* \rangle = \mathbb{E}^{\mathbb{Q}}\left(\frac{V_1^{\mathbf{h}}}{1+r}\right) \quad (22)$$

$$(\mathbf{V}_1^{\mathbf{h}})^* = \underbrace{\begin{pmatrix} 1 & \frac{S_1^1(\omega_1)}{1+r} & ... & \frac{S_1^N(\omega_1)}{1+r} & \\ 1 & \frac{S_1^1(\omega_2)}{1+r} & ... & \frac{S_1^N(\omega_2)}{1+r} & \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \frac{S_1^1(\omega_K)}{1+r} & ... & \frac{S_1^N(\omega_K)}{1+r} & \end{pmatrix}}_{=:\mathbf{A}^*} \mathbf{h} \quad (23)$$
$$= \mathbf{A}^*\mathbf{h} \quad (24)$$

Zustandspreise

$$\mathbf{A}^T \boldsymbol{\psi} = \bar{\mathbf{S}}_0 \quad (25)$$

bzw.

$$S_0^i = \sum_{k=1}^K S_1^i(\omega_k)\psi(\omega_k) \quad (26)$$
$$1 = \sum_{k=1}^K R_1^f \psi(\omega_k) \quad (27)$$

Stochastische Diskontfaktoren

$$1 = \sum_{k=1}^K \mathbb{P}(\omega_k) m(\omega_k) R_1^f = \mathbb{E}^{\mathbb{P}}(\mathbf{m}R_1^f), \quad (28)$$
$$S_0^i = \sum_{k=1}^K \mathbb{P}(\omega_k) m(\omega_k) S_1^i(\omega_k) = \mathbb{E}^{\mathbb{P}}(\mathbf{m}\mathbf{S}_1^i). \quad (29)$$

$$\mathbb{E}^{\mathbb{P}}(\mathbf{m}) = \frac{1}{R^f} = Z. \quad (30)$$

Zustandspreisdichte

$$1 = \sum_{k=1}^{K} \mathbb{P}(\omega_k) d(\omega_k) = \mathbb{E}^{\mathbb{P}}(\mathbf{d}), \tag{31}$$

$$S_0^i = \sum_{k=1}^{K} \mathbb{P}(\omega_k) d(\omega_k) \frac{\mathbf{S}_1^i(\omega_k)}{R^f} = \mathbb{E}^{\mathbb{P}} \left(\mathbf{d} \, \frac{\mathbf{S}_1^i}{R^f} \right). \tag{32}$$

Umrechnungsformel, $\boldsymbol{\psi}$ gegeben

$$m(\omega_i) = \frac{\psi(\omega_i)}{\mathbb{P}(\omega_i)} \tag{33}$$

$$\mathbb{Q}(\omega_i) = \psi(\omega_i) R_1^f \tag{34}$$

$$d(\omega_k) = \frac{\psi(\omega_i)}{\mathbb{P}(\omega_i)} R^f \tag{35}$$

Umrechnungsformel, $\mathbb{Q}$ gegeben

$$m(\omega_i) = \frac{\mathbb{Q}(\omega_i)}{\mathbb{P}(\omega_i) R_1^f} \tag{36}$$

$$\psi(\omega_i) = \frac{\mathbb{Q}(\omega_i)}{R_1^f} \tag{37}$$

$$d(\omega_k) = \frac{\mathbb{Q}(\omega_k)}{\mathbb{P}(\omega_k)} \tag{38}$$

Umrechnungsformel, $\mathbf{m}$ gegeben

$$\psi(\omega_i) = \mathbb{P}(\omega_i) m(\omega_i) \tag{39}$$

$$\mathbb{Q}(\omega_i) = \mathbb{P}(\omega_i) m(\omega_i) R_1^f \tag{40}$$

$$d(\omega_k) = m(\omega_k) R^f \tag{41}$$

Umrechnungsformel, $\mathbf{d}$ gegeben

$$\mathbb{Q}(\omega_k) = \mathbb{P}(\omega_k) d(\omega_k) \tag{42}$$

$$m(\omega_k) = \frac{d(\omega_k)}{R^f} \tag{43}$$

$$\psi(\omega_k) = \frac{\mathbb{P}(\omega_k) d(\omega_k)}{R^f} \tag{44}$$

cov-**Risikoabschlag**

$$\mathbf{S}_0^i = \mathbb{E}^{\mathbb{P}}(\mathbf{m}\mathbf{S}_1^i) = \mathbb{E}^{\mathbb{P}}(\mathbf{m}) \mathbb{E}^{\mathbb{P}}(\mathbf{S}_1^i) + \operatorname{cov}(\mathbf{m}, \mathbf{S}_1^i) \tag{45}$$

$$= Z \cdot \mathbb{E}^{\mathbb{P}}(\mathbf{S}_1^i) + \operatorname{cov}(\mathbf{m}, \mathbf{S}_1^i) \tag{46}$$

$$= \frac{1}{R^f} \cdot \mathbb{E}^{\mathbb{P}}(\mathbf{S}_1^i) + \operatorname{cov}(\mathbf{m}, \mathbf{S}_1^i). \tag{47}$$

Give me five Übersicht

$$S_0^i = \sum_{k=1}^{K} \psi(\omega_k) S_1^i(\omega_k) \tag{48}$$

$$S_0^i = \sum_{k=1}^{K} \mathbb{P}(\omega_k) d(\omega_k) \frac{S_1^i(\omega_k)}{R^f} = \mathbb{E}^{\mathbb{P}} \left(\mathbf{d} \, \frac{\mathbf{S}_1^i}{R^f} \right) \tag{49}$$

$$S_0^i = \sum_{k=1}^{K} \mathbb{P}(\omega_k) m(\omega_k) S_1^i(\omega_k) = \mathbb{E}^{\mathbb{P}}(\mathbf{m}\mathbf{S}_1^i) \tag{50}$$

$$S_0^i = \sum_{k=1}^{K} \mathbb{Q}(\omega_k) \frac{S_1^i(\omega_k)}{R_1^f} = \mathbb{E}^{\mathbb{Q}} \left(\frac{\mathbf{S}_1^i}{R^f} \right) \tag{51}$$

$$S_0^i = \mathbb{E}^{\mathbb{P}} \left(\frac{\mathbf{S}_1^i}{R^f} \right) + \operatorname{cov}(\mathbf{m}, \mathbf{S}_1^i) \tag{52}$$

Bewertung mit Replikation

$$p_X = V_0^h = h_0 + h_1 S_0$$
$$\mathbf{A}\mathbf{h} = X$$

Bewertung mit Risikoneutralwahrscheinlichkeiten

$$p_X = \mathbb{E}^{\mathbb{Q}} \left(\frac{X}{1+r} \right) \tag{53}$$

β-**Darstellung**

$$\mathbb{E}^{\mathbb{P}}(\mathbf{R}) = \frac{1}{\mathbb{E}^{\mathbb{P}}(\mathbf{m})} + \beta_{\mathbf{R},\mathbf{m}} \lambda_{\mathbf{m}}, \tag{54}$$

$$\beta_{\mathbf{R},\mathbf{m}} := \frac{\operatorname{cov}(\mathbf{m}, \mathbf{R})}{\mathbb{V}^{\mathbb{P}}(\mathbf{m})}, \tag{55}$$

$$\lambda_{\mathbf{m}} := -\frac{\mathbb{V}^{\mathbb{P}}(\mathbf{m})}{\mathbb{E}^{\mathbb{P}}(\mathbf{m})}. \tag{56}$$

BBM

Martingalwahrscheinlichkeit BBM

$$\tilde{p} = \frac{1+r-d}{u-d} \tag{57}$$

$$\tilde{p} = \frac{e^{r\Delta t} - d}{u-d} \tag{58}$$

Kalibrierung BBM

$$\beta = \frac{1}{2} \left(e^{-rdt} + e^{(r+\sigma^2)dt} \right) \tag{59}$$

$$u = \beta + \sqrt{\beta^2 - 1} \tag{60}$$

$$d = 1/u \tag{61}$$

$$\tilde{p} = \frac{e^{r\Delta t} - d}{u-d} \tag{62}$$

Risikoneutralbewertung

$$V_0 = \left(\frac{1}{1+r} \right) \mathbb{E}^{\mathbb{Q}}(V_1) \tag{63}$$

Risikoneutralbewertung

$$V_0 = \frac{1}{(1+r)^T}\,\mathbb{E}^{\mathbb{Q}}(V_T) \tag{64}$$

Risikoneutralbewertung

$$V_t = \frac{1}{(1+r)}\,\mathbb{E}_t^{\mathbb{Q}}(V_{t+1}) \tag{65}$$

Auszahlungsprofil (Call/Put)

$$C_T = \max\{S_T - K, 0\} \tag{66}$$
$$P_T = \max\{K - S_T, 0\} \tag{67}$$

ZPBBM - Replikationsgleichungen

$$M_1(u)(1+r) + \Delta_1(u)S_2(u,u) = X_2(u,u) \tag{68}$$
$$M_1(u)(1+r) + \Delta_1(u)S_2(u,d) = X_2(u,d) \tag{69}$$
$$X_1(u) = M_1(u) + \Delta_1(u)S_1(u) \tag{70}$$

$$M_1(d)(1+r) + \Delta_1(d)S_2(d,u) = X_2(d,u) \tag{71}$$
$$M_1(d)(1+r) + \Delta_1(d)S_2(d,d) = X_2(d,d) \tag{72}$$
$$X_1(d) = M_1(d) + \Delta_1(d)S_1(d) \tag{73}$$

$$M_0(1+r) + \Delta_0 S_1(u) = X_1(u) \tag{74}$$
$$M_0(1+r) + \Delta_0 S_1(d) = X_1(d) \tag{75}$$
$$X_0 = M_0 + \Delta_0 S_0 \tag{76}$$

Entscheidungen unter Risiko

Sicherheitsäquivalent $c(\mathbb{P})$, Risikoprämie $\varrho(\mathbb{P})$

$$u(c(\mathbb{P})) = \mathbb{E}^{\mathbb{P}}(u) = U(\mathbb{P}) = \int u\,d\mathbb{P} \tag{77}$$
$$\varrho(\mathbb{P}) = m(\mathbb{P}) - c(\mathbb{P}) \tag{78}$$

Approximation Arrow-Pratt

$$\varrho \approx -\frac{1}{2}\frac{u''(m)}{u'(m)}\mathbb{V}(\mathbb{P}) \tag{79}$$

Koeffizient der absoluten und der relativen Risikoaversion

$$\alpha(x) = -\frac{u''(x)}{u'(x)}, \quad \rho(x) = \alpha(x)x \tag{80}$$

Optimales Portfolio

$$H(h_1^*) = \mathbb{E}[\,u(V_1)\,] \tag{81}$$
$$= \mathbb{E}[\,u((1+r)w_0 + h_1^*(R-r))\,] \tag{82}$$
$$H'(h_1^*) = \mathbb{E}[\,u'((1+r)w_0 + h_1^*(R-r))\cdot(R-r)\,] = 0 \tag{83}$$

Optimales Portfolio $u(s) = \ln(s)$

$$\frac{h_1^*}{w_0} = (1+r)\cdot\frac{\mathbb{E}(R-r)}{(R^u - r)(r - R^d)}. \tag{84}$$

Bewertungsgleichung

$$V_0 = \frac{1}{1+r}\mathbb{E}\left[\frac{u'}{\mathbb{E}(u')}V_1\right] = \frac{\mathbb{E}(mV_1)}{1+r} \tag{85}$$

Überschussrendite

$$\mathbb{E}(R) - r = \frac{-\mathrm{cov}(R,u')}{\mathbb{E}(u')} = \mathrm{cov}(R,m) \tag{86}$$

μ-σ-optimale Portfolio

Erwartete Rendite und Varianz

$$\mathbb{E}(\mathbf{R}) = \mathbf{w}^T\boldsymbol{\mu}, \mathbb{V}(\mathbf{R}) = \mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w} \tag{87}$$

Global Varianzminimales Portfolio

$$\mathbf{w} = \frac{\boldsymbol{\Sigma}^{-1}\mathbf{1}}{\mathbf{1}^T\boldsymbol{\Sigma}^{-1}\mathbf{1}} \tag{88}$$

Grenzhyperbel

$$\sigma^2 = \frac{A - 2B\mu_{\mathbf{w}} + C\mu_{\mathbf{w}}^2}{AC - B^2} \tag{89}$$
$$A = \boldsymbol{\mu}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}, B = \boldsymbol{\mu}^T\boldsymbol{\Sigma}^{-1}\mathbf{1}, C = \mathbf{1}^T\boldsymbol{\Sigma}^{-1}\mathbf{1}$$

Varianzminimales Portfolio (ohne Geldmarkt)

$$\mathbf{w}^* = \boldsymbol{\Sigma}^{-1}\mathbf{M}\mathbf{B}^{-1}\tilde{\boldsymbol{\mu}}^*, \text{ mit} \tag{90}$$

$$\mathbf{M} = (\boldsymbol{\mu},\ \mathbf{1}) \tag{91}$$
$$\tilde{\boldsymbol{\mu}}^* = (\mu^*,\ 1)^T \tag{92}$$
$$\boldsymbol{B} = \boldsymbol{M}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{M} \tag{93}$$

Portfoliorendite und Sharpe-Quotient

$$\mathbb{E}(R^{\tilde{\mathbf{w}}}) = R^f \pm \left[\frac{\mathbb{E}(R^{\mathbf{w}}) - R^f}{\sigma_{R^{\mathbf{w}}}}\right]\cdot\sigma_{R^{\tilde{\mathbf{w}}}} \tag{94}$$

Tangentialportfolio

$$\mathbf{w}^{ta} = \frac{1}{\mathbf{1}^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - R^f\mathbf{1})}\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - R^f\mathbf{1}) \tag{95}$$

Varianzminimales Portfolio (einschl. Geldmarkt)

$$\tilde{\mathbf{w}}^* = \frac{\mu^* - R^f}{(\boldsymbol{\mu} - R^f\mathbf{1})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - R^f\mathbf{1})}\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - R^f\mathbf{1}) \tag{96}$$

$$(1-\alpha)R^f + \alpha\mathbb{E}(R^{\mathrm{wta}}) = \mu^*, \tag{97}$$
$$\alpha = \frac{\mu^* - R^f}{\mathbb{E}(R^{\mathrm{wta}}) - R^f} \tag{98}$$
$$\tilde{\mathbf{w}}^* = \alpha\mathbf{w}^{ta} \tag{99}$$
$$\mathbb{V}(R^{\tilde{w}}) = \alpha^2\mathbb{V}(R^{ta}) \tag{100}$$

Fehler gefunden? Ich bin für Hinweise sehr dankbar: jaegera ät htw-berlin punkt de