

Formelsammlung für Finanzmathematik

Manfred Jäger-Ambrozewicz, <https://mathfred.de/> und <https://mathstat.de/>

Stand: 5. Juli 2026

EPFMM

Anschaffungskosten EPFMM

$$V_0^{\mathbf{h}} = h_0 + S_0^1 h_1 + \dots S_0^N h_N \quad (1)$$

$$= (1, \mathbf{S}_0^T) \mathbf{h} \quad (2)$$

$$= \mathbf{h}^T \begin{pmatrix} 1 \\ S_0^1 \\ \vdots \\ S_0^N \end{pmatrix} \quad (3)$$

$$= \mathbf{h}^T \begin{pmatrix} 1 \\ \mathbf{S}_0 \end{pmatrix} = \langle \mathbf{h}, \begin{pmatrix} 1 \\ \mathbf{S}_0 \end{pmatrix} \rangle \quad (4)$$

$$= \langle \mathbf{h}, \bar{\mathbf{S}}_0 \rangle, \quad (5)$$

Auszahlung EPFMM

$$\mathbf{V}_1^{\mathbf{h}} = \mathbf{A} \mathbf{h}. \quad (6)$$

Auszahlungsmatrix

$$\mathbf{A} = \begin{pmatrix} 1+r & S_1^1(\omega_1) & \dots & S_1^N(\omega_1) \\ 1+r & S_1^1(\omega_2) & \dots & S_1^N(\omega_2) \\ \vdots & \vdots & \vdots & \vdots \\ 1+r & S_1^1(\omega_K) & \dots & S_1^N(\omega_K) \end{pmatrix}.$$

Arbitrage

$$V_0^{\mathbf{h}} = 0, 0 \neq \mathbf{V}_1^{\mathbf{h}} \geq \mathbf{0} \quad (7)$$

bzw.

$$V_0^{\mathbf{h}} = 0, \mathbf{V}_1^{\mathbf{h}} \geq \mathbf{0} \text{ und} \quad (8)$$

$$V_1^{\mathbf{h}}(\omega) > 0 \text{ für mindestens ein } \omega \in \Omega. \quad (9)$$

Risikoneutralwahrscheinlichkeit

$$\mathbb{Q}(\omega) > 0 \quad (10)$$

$$S_0^i = \mathbb{E}^{\mathbb{Q}} \left(\frac{S_1^i}{1+r} \right) \quad (11)$$

$$= \sum_{k=1}^K q_k \frac{S_1^i(\omega_k)}{1+r} = \sum_{k=1}^K q_k \cdot S_1^{i*}(\omega_k) \quad (12)$$

$$\mathbf{S}_0 = (\mathbf{S}_1^*)^T \mathbf{q} \quad (13)$$

$$q_i = q(\omega_i) = \mathbb{Q}(\omega_i) \quad (14)$$

$$\mathbf{S}_1^* = \begin{pmatrix} \frac{S_1^1(\omega_1)}{1+r} & \dots & \frac{S_1^{N-1}(\omega_1)}{1+r} & \frac{S_1^N(\omega_1)}{1+r} \\ \frac{S_1^1(\omega_2)}{1+r} & \dots & \frac{S_1^{N-1}(\omega_2)}{1+r} & \frac{S_1^N(\omega_2)}{1+r} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{S_1^1(\omega_K)}{1+r} & \dots & \frac{S_1^{N-1}(\omega_K)}{1+r} & \frac{S_1^N(\omega_K)}{1+r} \end{pmatrix} \quad (15)$$

Risikoneutralwahrscheinlichkeit

$$\mathbf{S}_0 = (\mathbf{S}_1^*)^T \mathbf{q}, \sum_{i=1}^K q_i = 1, \mathbf{q} > \mathbf{0}. \quad (16)$$

$$\bar{\mathbf{S}}_0 = (\mathbf{A}^*)^T \mathbf{q}, \mathbf{q} > \mathbf{0} \quad (17)$$

$$\mathbf{A}^* = \begin{pmatrix} 1 & \frac{S_1^1(\omega_1)}{1+r} & \dots & \frac{S_1^{N-1}(\omega_1)}{1+r} & \frac{S_1^N(\omega_1)}{1+r} \\ 1 & \frac{S_1^1(\omega_2)}{1+r} & \dots & \frac{S_1^{N-1}(\omega_2)}{1+r} & \frac{S_1^N(\omega_2)}{1+r} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \frac{S_1^1(\omega_K)}{1+r} & \dots & \frac{S_1^{N-1}(\omega_K)}{1+r} & \frac{S_1^N(\omega_K)}{1+r} \end{pmatrix} \quad (18)$$

Risikoneutralwahrscheinlichkeit

$$V_0^{\mathbf{h}} = \sum_{k=1}^K q(\omega_k) \frac{V_1^{\mathbf{h}}(\omega_k)}{1+r} \quad (19)$$

$$= \sum_{k=1}^K q(\omega_k) (V_1^{\mathbf{h}})^*(\omega_k) \quad (20)$$

$$= ((\mathbf{V}_1^{\mathbf{h}})^*)^T \mathbf{q} = \mathbf{q}^T (\mathbf{V}_1^{\mathbf{h}})^* \quad (21)$$

$$= \langle \mathbf{q}, (\mathbf{V}_1^{\mathbf{h}})^* \rangle = \mathbb{E}^{\mathbb{Q}} \left(\frac{V_1^{\mathbf{h}}}{1+r} \right) \quad (22)$$

$$(\mathbf{V}_1^{\mathbf{h}})^* = \underbrace{\begin{pmatrix} 1 & \frac{S_1^1(\omega_1)}{1+r} & \dots & \frac{S_1^N(\omega_1)}{1+r} \\ 1 & \frac{S_1^1(\omega_2)}{1+r} & \dots & \frac{S_1^N(\omega_2)}{1+r} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \frac{S_1^1(\omega_K)}{1+r} & \dots & \frac{S_1^N(\omega_K)}{1+r} \end{pmatrix}}_{=: \mathbf{A}^*} \mathbf{h} \quad (23)$$

$$= \mathbf{A}^* \mathbf{h} \quad (24)$$

Zustandspreise

$$\mathbf{A}^T \boldsymbol{\psi} = \bar{\mathbf{S}}_0 \quad (25)$$

bzw.

$$S_0^i = \sum_{k=1}^K S_1^i(\omega_k) \psi(\omega_k) \quad (26)$$

$$1 = \sum_{k=1}^K R_1^f(\omega_k) \psi(\omega_k) \quad (27)$$

Stochastische Diskontfaktoren

$$1 = \sum_{k=1}^K \mathbb{P}(\omega_k) m(\omega_k) R_1^f = \mathbb{E}^{\mathbb{P}}(\mathbf{m} R_1^f), \quad (28)$$

$$S_0^i = \sum_{k=1}^K \mathbb{P}(\omega_k) m(\omega_k) S_1^i(\omega_k) = \mathbb{E}^{\mathbb{P}}(\mathbf{m} \mathbf{S}_1^i). \quad (29)$$

$$\mathbb{E}^{\mathbb{P}}(\mathbf{m}) = \frac{1}{R^f} = Z. \quad (30)$$

Zustandspreisdichte

$$1 = \sum_{k=1}^K \mathbb{P}(\omega_k) d(\omega_k) = \mathbb{E}^{\mathbb{P}}(\mathbf{d}), \quad (31)$$

$$S_0^i = \sum_{k=1}^K \mathbb{P}(\omega_k) d(\omega_k) \frac{\mathbf{S}_1^i(\omega_k)}{R^f} = \mathbb{E}^{\mathbb{P}}\left(\mathbf{d} \frac{\mathbf{S}_1^i}{R^f}\right). \quad (32)$$

Umrechnungsformel, ψ gegeben

$$m(\omega_i) = \frac{\psi(\omega_i)}{\mathbb{P}(\omega_i)} \quad (33)$$

$$\mathbb{Q}(\omega_i) = \psi(\omega_i) R_1^f \quad (34)$$

$$d(\omega_k) = \frac{\psi(\omega_k)}{\mathbb{P}(\omega_k)} R^f \quad (35)$$

Umrechnungsformel, \mathbb{Q} gegeben

$$m(\omega_i) = \frac{\mathbb{Q}(\omega_i)}{\mathbb{P}(\omega_i) R_1^f} \quad (36)$$

$$\psi(\omega_i) = \frac{\mathbb{Q}(\omega_i)}{R_1^f} \quad (37)$$

$$d(\omega_k) = \frac{\mathbb{Q}(\omega_k)}{\mathbb{P}(\omega_k)} \quad (38)$$

Umrechnungsformel, \mathbf{m} gegeben

$$\psi(\omega_i) = \mathbb{P}(\omega_i) m(\omega_i) \quad (39)$$

$$\mathbb{Q}(\omega_i) = \mathbb{P}(\omega_i) m(\omega_i) R_1^f \quad (40)$$

$$d(\omega_k) = m(\omega_k) R^f \quad (41)$$

Umrechnungsformel, \mathbf{d} gegeben

$$\mathbb{Q}(\omega_k) = \mathbb{P}(\omega_k) d(\omega_k) \quad (42)$$

$$m(\omega_k) = \frac{d(\omega_k)}{R^f} \quad (43)$$

$$\psi(\omega_k) = \frac{\mathbb{P}(\omega_k) d(\omega_k)}{R^f} \quad (44)$$

cov-Risikoabschlag

$$\mathbf{S}_0^i = \mathbb{E}^{\mathbb{P}}(\mathbf{m} \mathbf{S}_1^i) = \mathbb{E}^{\mathbb{P}}(\mathbf{m}) \mathbb{E}^{\mathbb{P}}(\mathbf{S}_1^i) + \text{cov}(\mathbf{m}, \mathbf{S}_1^i) \quad (45)$$

$$= Z \cdot \mathbb{E}^{\mathbb{P}}(\mathbf{S}_1^i) + \text{cov}(\mathbf{m}, \mathbf{S}_1^i) \quad (46)$$

$$= \frac{1}{R^f} \cdot \mathbb{E}^{\mathbb{P}}(\mathbf{S}_1^i) + \text{cov}(\mathbf{m}, \mathbf{S}_1^i). \quad (47)$$

Magic Cube Übersicht

$$p_{\mathbf{X}} = V_0^{\mathbf{h}}$$

$$p_{\mathbf{X}} = \sum_{k=1}^K \psi(\omega_k) \mathbf{X}(\omega_k)$$

$$p_{\mathbf{X}} = \sum_{k=1}^K \mathbb{P}(\omega_k) d(\omega_k) \frac{\mathbf{X}(\omega_k)}{R^f} = \mathbb{E}^{\mathbb{P}}\left(\mathbf{d} \frac{\mathbf{X}}{R^f}\right)$$

$$p_{\mathbf{X}} = \sum_{k=1}^K \mathbb{P}(\omega_k) m(\omega_k) \mathbf{X}(\omega_k) = \mathbb{E}^{\mathbb{P}}(\mathbf{m} \mathbf{X})$$

$$p_{\mathbf{X}} = \sum_{k=1}^K \mathbb{Q}(\omega_k) \frac{\mathbf{X}(\omega_k)}{R^f} = \mathbb{E}^{\mathbb{Q}}\left(\frac{\mathbf{X}}{R^f}\right)$$

$$p_{\mathbf{X}} = \mathbb{E}^{\mathbb{P}}\left(\frac{\mathbf{X}}{R^f}\right) + \text{cov}(\mathbf{m}, \mathbf{X})$$

Bewertung mit Replikation

$$p_X = V_0^{\mathbf{h}} = h_0 + h_1 S_0$$

$$\mathbf{A} \mathbf{h} = X$$

Bewertung mit Risikoneutralwahrscheinlichkeiten

$$p_X = \mathbb{E}^{\mathbb{Q}}\left(\frac{X}{1+r}\right) \quad (48)$$

β -Darstellung

$$\mathbb{E}^{\mathbb{P}}(\mathbf{R}) = \frac{1}{\mathbb{E}^{\mathbb{P}}(\mathbf{m})} + \beta_{\mathbf{R}, \mathbf{m}} \lambda_{\mathbf{m}}, \quad (49)$$

$$\beta_{\mathbf{R}, \mathbf{m}} := \frac{\text{cov}(\mathbf{m}, \mathbf{R})}{\mathbb{V}^{\mathbb{P}}(\mathbf{m})}, \quad (50)$$

$$\lambda_{\mathbf{m}} := -\frac{\mathbb{V}^{\mathbb{P}}(\mathbf{m})}{\mathbb{E}^{\mathbb{P}}(\mathbf{m})}. \quad (51)$$

BBM

Martingalwahrscheinlichkeit BBM

$$\tilde{p} = \frac{1+r-d}{u-d} \quad (52)$$

$$\tilde{p} = \frac{e^{r\Delta t} - d}{u-d} \quad (53)$$

Kalibrierung BBM

$$\beta = \frac{1}{2} \left(e^{-rdt} + e^{(r+\sigma^2)dt} \right) \quad (54)$$

$$u = \beta + \sqrt{\beta^2 - 1} \quad (55)$$

$$d = 1/u \quad (56)$$

$$\tilde{p} = \frac{e^{r\Delta t} - d}{u-d} \quad (57)$$

Risikoneutralbewertung

$$V_0 = \left(\frac{1}{1+r} \right) \mathbb{E}^{\mathbb{Q}}(V_1) \quad (58)$$

Risikoneutralbewertung

$$V_0 = \frac{1}{(1+r)^T} \mathbb{E}^{\mathbb{Q}}(V_T) \quad (59)$$

Risikoneutralbewertung

$$V_t = \frac{1}{(1+r)} \mathbb{E}_t^{\mathbb{Q}}(V_{t+1}) \quad (60)$$

Auszahlungsprofil (Call/Put)

$$C_T = \max\{S_T - K, 0\} \quad (61)$$

$$P_T = \max\{K - S_T, 0\} \quad (62)$$

ZPBMM - Replikationsgleichungen

$$M_1(u)(1+r) + \Delta_1(u)S_2(u, u) = X_2(u, u) \quad (63)$$

$$M_1(u)(1+r) + \Delta_1(u)S_2(u, d) = X_2(u, d) \quad (64)$$

$$X_1(u) = M_1(u) + \Delta_1(u)S_1(u) \quad (65)$$

$$M_1(d)(1+r) + \Delta_1(d)S_2(d, u) = X_2(d, u) \quad (66)$$

$$M_1(d)(1+r) + \Delta_1(d)S_2(d, d) = X_2(d, d) \quad (67)$$

$$X_1(d) = M_1(d) + \Delta_1(d)S_1(d) \quad (68)$$

$$M_0(1+r) + \Delta_0 S_1(u) = X_1(u) \quad (69)$$

$$M_0(1+r) + \Delta_0 S_1(d) = X_1(d) \quad (70)$$

$$X_0 = M_0 + \Delta_0 S_0 \quad (71)$$

Entscheidungen unter Risiko**Sicherheitsäquivalent $c(\mathbb{P})$, Risikoprämie $\varrho(\mathbb{P})$**

$$u(c(\mathbb{P})) = \mathbb{E}^{\mathbb{P}}(u) = U(\mathbb{P}) = \int u d\mathbb{P} \quad (72)$$

$$\varrho(\mathbb{P}) = m(\mathbb{P}) - c(\mathbb{P}) \quad (73)$$

Approximation Arrow-Pratt

$$\varrho \approx -\frac{1}{2} \frac{u''(m)}{u'(m)} \mathbb{V}(\mathbb{P}) \quad (74)$$

Koeffizient der absoluten und der relativen Risikoaversion

$$\alpha(x) = -\frac{u''(x)}{u'(x)}, \quad \rho(x) = \alpha(x)x \quad (75)$$

Optimales Portfolio

$$H(h_1) = \mathbb{E}[u(V_1)] \quad (76)$$

$$= \mathbb{E}[u((1+r)w_0 + h_1^*(R-r))] \quad (77)$$

$$H'(h_1) = \mathbb{E}[u'((1+r)w_0 + h_1^*(R-r)) \cdot (R-r)] \quad (78)$$

$$= 0 \quad (78)$$

Optimales Portfolio $u(s) = \ln(s)$

$$\frac{h_1^*}{w_0} = (1+r) \cdot \frac{\mathbb{E}(R-r)}{(R^u - r)(r - R^d)} \quad (79)$$

Bewertungsgleichung, stochastischer Diskontfaktor

$$V_0 = \mathbb{E} \left[\frac{u'}{\mathbb{E}(u')} \frac{V_1}{1+r} \right] = \frac{\mathbb{E}(mV_1)}{1+r} \quad (80)$$

$$m = \frac{u'}{\mathbb{E}(u')} \quad (81)$$

Überschussrendite

$$\mathbb{E}(R) - r = \frac{-\text{cov}(R, u')}{\mathbb{E}(u')} = \text{cov}(R, m) \quad (82)$$

 μ - σ -optimale Portfolio**Erwartete Rendite und Varianz**

$$\mathbb{E}(\mathbf{R}) = \mathbf{w}^T \boldsymbol{\mu}, \quad \mathbb{V}(\mathbf{R}) = \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} \quad (83)$$

Global Varianzminimales Portfolio

$$\mathbf{w} = \frac{\boldsymbol{\Sigma}^{-1} \mathbf{1}}{\mathbf{1}^T \boldsymbol{\Sigma}^{-1} \mathbf{1}} \quad (84)$$

Grenzhyperbel

$$\sigma^2 = \frac{A - 2B\mu_{\mathbf{w}} + C\mu_{\mathbf{w}}^2}{AC - B^2} \quad (85)$$

$$A = \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}, \quad B = \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \mathbf{1}, \quad C = \mathbf{1}^T \boldsymbol{\Sigma}^{-1} \mathbf{1}$$

Varianzminimales Portfolio (ohne Geldmarkt)

$$\mathbf{w}^* = \boldsymbol{\Sigma}^{-1} \mathbf{M} \mathbf{B}^{-1} \tilde{\boldsymbol{\mu}}^*, \quad \text{mit} \quad (86)$$

$$\mathbf{M} = (\boldsymbol{\mu}, \mathbf{1}) \quad (87)$$

$$\tilde{\boldsymbol{\mu}}^* = (\mu^*, \mathbf{1})^T \quad (88)$$

$$\mathbf{B} = \mathbf{M}^T \boldsymbol{\Sigma}^{-1} \mathbf{M} \quad (89)$$

Portfoliorendite und Sharpe-Quotient

$$\mathbb{E}(R^{\bar{\mathbf{w}}}) = R^f \pm \left[\frac{\mathbb{E}(R^{\mathbf{w}}) - R^f}{\sigma_{R^{\mathbf{w}}}} \right] \cdot \sigma_{R^{\bar{\mathbf{w}}}} \quad (90)$$

Tangentialportfolio

$$\mathbf{w}^{ta} = \frac{1}{\mathbf{1}^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu} - R^f \mathbf{1})} \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu} - R^f \mathbf{1}) \quad (91)$$

Varianzminimales Portfolio (einschl. Geldmarkt)

$$\tilde{\mathbf{w}}^* = \frac{\mu^* - R^f}{(\boldsymbol{\mu} - R^f \mathbf{1})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu} - R^f \mathbf{1})} \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu} - R^f \mathbf{1}) \quad (92)$$

$$(1 - \alpha)R^f + \alpha \mathbb{E}(R^{\mathbf{w}^{ta}}) = \mu^*, \quad (93)$$

$$\alpha = \frac{\mu^* - R^f}{\mathbb{E}(R^{\mathbf{w}^{ta}}) - R^f} \quad (94)$$

$$\tilde{\mathbf{w}}^* = \alpha \mathbf{w}^{ta} \quad (95)$$

$$\mathbb{V}(R^{\tilde{\mathbf{w}}}) = \alpha^2 \mathbb{V}(R^{\mathbf{w}^{ta}}) \quad (96)$$

Fehler gefunden? Ich bin für Hinweise sehr dankbar: jaegera@htw-berlin.punkt.de